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# Historical Survey on Enhanced Mixing in Scramjet Engines

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**Select methods are reviewed that have been used by the scramjet community to enhance mixing of fuel with oxidizer to achieve high combustion efficiency with reduced length combustors. The review also includes research on enhanced supersonic free shear layer mixing from the jet noise reduction community. This latter community has considered concepts involving acoustic excitation or passive excitation of shear layer instabilities to minimize noise production. Several of these concepts are amendable to active control for system optimization.**

## Nomenclature

| | | |
|---|---|---|
| $b$ | = | $R_{0.5} - h$ |
| $C_\delta, (C_\delta)_0$ | = | compressible and incompressible vorticity thickness [Eq. (1)] |
| $c_1, c_2$ | = | sound speed in high and low speed coflow |
| $H$ | = | radius of potential core |
| $M_C$ | = | convective Mach number [Eq. (2)] |
| $p'$ | = | fluctuating pressure |
| $U, U_1, U_2$ | = | streamwise velocity/velocities of high and low speed co-flow |
| $u', v', w'$ | = | turbulent velocity components |
| $\alpha_r, \alpha_i$ | = | real and imaginary part of instability streamwise wavenumber |
| $\alpha_1$ | = | adjustable parameter in Eq. (4) |
| $\delta$ | = | shear layer mixing width |
| $\varepsilon_{ij}, \varepsilon_s$ | = | compressible and incompressible turbulent dissipation |
| $\eta$ | = | nondimensional shear layer coordinate, $(R-h)/b$ |
| $\rho_1, \rho_2$ | = | densities of high- and low-speed coflow |
| $\omega$ | = | radian frequency |

## Introduction

THE U.S. government has investigated scramjet engine development for over 40 years. One important aspect of this technical development is the successful demonstration of an efficient combustion system. To achieve satisfactory vehicle airplane thrust to weight ratios, the scramjet engine requires a close coupled system where burning can take place in the exhaust nozzle. To accomplish this, the engine is designed to be combustion choked. Thus, the combustion process must take place rapidly and ensure flame-holding capability with short fuel residence times. At supersonic fuel injection and oxidizer speeds, a concept is desired that can rapidly mix fuel and oxidizer while permitting sufficient fuel residence times to achieve flammability. This process must be coupled with the engine inlet system, where the pressure rise due to the combustion process does not lead to engine unstart.

This paper presents the results of a review conducted to survey previous technology associated with scramjet engine development. Only those portions of this survey that may relate to a scramjet combustor are considered. The purpose of the survey was to determine which of the numerous concepts previously investigated hold merit for future consideration. Controlled mixing enhancement of supersonic–subsonic streams with different gas density is also important regarding the reduction of supersonic jet noise. Several concepts from this area of technology are included for their value to successful scramjet engine development.

## Nonreacting Compressible Free Shear Layer Mixing

The mixing rate for nonreacting turbulent free shear layers of two different gas species is one area that has been extensively studied. Past experimental,[1–4] theoretical,[5,6] and numerical[7,8] studies have clearly shown a rapid decrease in mixing efficiency as Mach number is increased into the supersonic regime. The vortex-pairing model proposed by Winant and Browand[9] is most likely not applicable because it applies to extremely low Reynolds number shear layers or in the very initial region of shear layer origin.

The majority of the previous studies have been conducted using the parallel stream mixing layer geometry. Brown and Roshko[2] attempted to explain the effect of density on decreased mixing efficiency with Mach number. They studied the problem with low subsonic parallel streams whose velocity ratio varied between $0.14 < U_2/U_1 < 0.38$ and density ratio between $0.14 < \rho_2/\rho_1 < 7$. Their results, which were shown in terms of a vorticity thickness,

$$C_\delta = \frac{U_1 - U_2}{(\partial U/\partial y)\max} \tag{1}$$

showed little influence of density ratio on decreased mixing efficiency. However, their studies clearly demonstrated the existence of large-scale turbulence structure in the shear layer that could be related to linear stability theory.

Papamouschou and Roshko[3] extended their study to supersonic pressure balanced parallel streams and utilized concepts from the Gropengiesser[10] application of linear stability theory to relate the decreased mixing efficiency to a convective Mach number. Their[3] test covered the range where the velocity and density ratio, respectively, varied from $0.04 < U_2/U_1 < 0.93$ and $0.24 < \rho_2/\rho_1 < 9.2$ with stream Mach numbers from 0.2 to 3.4. Their argument to explain shear layer spread rates was that mixing occurs at a rate consistent with the growth of turbulent large-scale structure in a frame of reference at which the large scales are convected. Their analysis showed that the convective Mach number number $M_C$ could be expressed by

$$M_C = \frac{U_1}{c_1}\left\{\frac{\left[1 + (U_2/U_1)\sqrt{\rho_2/\rho_1}\right]}{\left(1 + \sqrt{\rho_2/\rho_1}\right)}\right\} = 2\frac{(U_1 - U_2)}{(c_1 + c_2)} \tag{2}$$

where $c_1$ and $c_2$ are the respective speeds of sound in each stream. Papamouschou and Roshko[3] then explicitly related reduced shear

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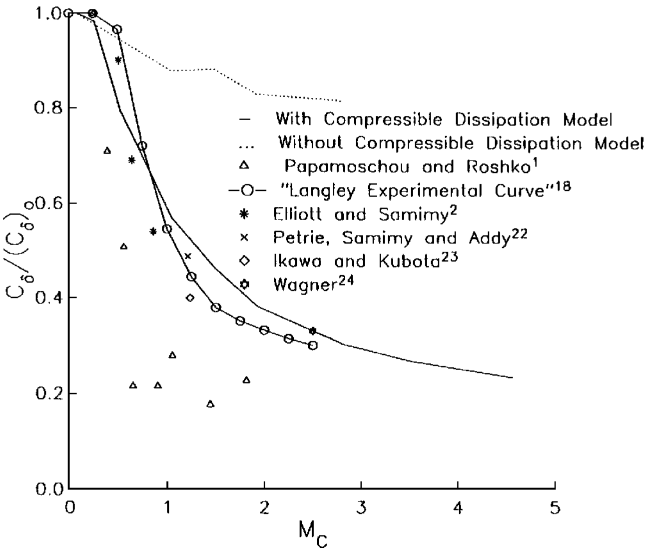



**Fig. 1 Normalized shear layer growth with convective Mach number (from Sarkar and Balakrishnan[11]).**

layer growth at compressible speed to incompressible shear layer growth as follows:

$$\frac{C_\delta}{(C_\delta)_0} = \frac{C_\delta c_1 M_C}{\mathrm{const}(U_1 - U_2)} \tag{3}$$

where $C_\delta = \mathrm{d}\delta/\mathrm{d}x$ and $(C_\delta)_0$ are, respectively, the measured shear layer growth rate and incompressible growth rate at the same velocity and density ratios. The constant in the preceding expression is 0.17 for measurement by visualization and 0.14 for measurement by pitot tube. Figure 1, from Sarkar and Balakrishnan,[11] clearly shows the effect compressibility has on mixing efficiency. Plotted in Fig. 1 is the NASA Langley Research Center experimental curve from Birch and Eggers[1] along with data from several other investigators including Papamouschou and Roshko.[3] Sarkar and Balakrishnan[11] attributed decreased mixing efficiency by neglect of the fluctuating dilatation in the pressure–strain correlation that appears in the turbulent kinetic energy transport equation of standard two-equation Reynolds-averaged Navier–Stokes flow solvers. Their analysis led to the introduction of a compressible dissipation term so that the total dissipation is given by

$$\varepsilon_{ij} = \tfrac{2}{3}\bar{\rho}\varepsilon S\left(1 + \alpha_1 M_t^2\right)\delta_{ij} \tag{4}$$

where $M_t$, $\bar{\rho}$, and $\delta_{ij}$ are turbulent Mach number, mean density, and Kronecker delta function, respectively. The effect of compressible dissipation that Sarkar and Balakrishnan[11] obtained for numerical prediction of shear layer growth rate with convective Mach number is shown in Fig. 1. As can be seen, their approach comes close to matching experimental data for $\alpha_1 = 1$.

The large eddy simulations (LES) conducted by Childs et al.[8] show that at high convective Mach number the turbulent structure becomes much more anisotropic with much larger streamwise normal stresses than the incompressible counterpart. This they denoted as swept structure to indicate much stronger streamwise vorticity, as have Lu and Lele.[6] The LES simulations by Childs et al.[8] shows that the pressure–axial velocity correlation is less effective at transferring energy to spanwise structure as the convective Mach number increases. The experimental data of Goebel and Dutton[4] confirm these findings in that they measured reduced spanwise turbulence intensities and normalized kinematic Reynolds stress $u'v'/\sqrt{u'^2}\sqrt{v'^2}$ with increasing convective Mach number.

Morris et al.[5] using linear stability theory solved the compressible Rayleigh model and obtained excellent agreement with the data of Papamouschou and Roshko.[3] In their application of the theory they,[5] ignored any resonant behavior among large-scale instability mode structure and suggested that each large-scale structure independently extracts energy from the mean flow. Their concept also included an eddy viscosity model to handle dissipation of energy from large to small scales. Their analysis shows that shear layer growth can be linked to amplitude growth of the large-scale structure. Their analysis showed that the spread rate could be analytically expressed as

$$C_\delta = \frac{4\delta}{(1 - r^2)[rI_3 + (1 - r)I_4]} \times \int_0^\infty \int_{-\infty}^\infty \alpha_i(\omega, \beta) A^2(x : \omega, \beta)\,\mathrm{d}\beta\,\mathrm{d}\omega$$

$$I_3 = \int_{-\infty}^\infty gh(1 - h)\,\mathrm{d}\eta, \qquad I_4 = \int_{-\infty}^\infty gh^2(1 - h)\,\mathrm{d}\eta \tag{5}$$

where $\alpha_i$ is the instability wave growth rate, A is the amplitude of the instability wave, and $\beta$ is the spanwise wave number. The velocity ratio is $r = U_2/U_1$. The density ratio is included in the term $g$, which is Crocco's relation for mean density. The analysis is relevant to the hyperbolic tangent profile given by $h(\eta) = 0.5(1 + \tanh\eta)$. The analysis by Morris et al.[5] compares favorably to data to convective Mach numbers of 1.2. However, the analysis is for two-dimensional flow, and based on the swept structure results of Childs et al.,[8] one would expect that the linear stability analysis would have to be applied three dimensionally to capture the strong anisotropic behavior at high speed.

In addition to three-dimensional effects, flows from nozzles, such as scramjet fuel injectors, will have at least two distinct shear layers that can interact as the instability waves convect and grow in the streamwise direction. Under these circumstances, even the convective Mach number of large-scale structure is affected by this interaction. Consider the heated supersonic flow from a single rectangular jet in a duct with subsonic coflow as shown in the focused schlieren record of Fig. 2. Here the primary flow has a pressure ratio of 2.51 and total temperature of 610 K, and the secondary flow has a pressure ratio of 1.40 and total temperature of 312 K. These data were acquired using a small-scale jet model. We were interested in determining fundamental differences in mixing between splitter plates and rectangular nozzles. The compressible Rayleigh model was applied, and the phase speed of the most highly amplified instability wave at peak growth was taken as one to determine convective Mach number. Table 1 shows a summary of the analysis for three cases run with the geometry of Fig. 3. Note the primary stream involves both heated and nonheated flow. As can be seen, excellent agreement is obtained from linear stability theory and that from Papamouschou and Roshko,[3] [(P&R) in Table 1]. However, the most amplified mode from the rectangular jet indicates that it travels much faster for the rectangular jet whether exhausting to free space or into a duct as in Fig. 2.

It is important to mention the work of Huerre and Monkewitz,[12] who investigated absolute and convective instabilities in free shear layers. The linear instability wave model is generally cast as spatial stability theory. However, under certain circumstances where self-excited resonance is possible, temporal instability theory is more

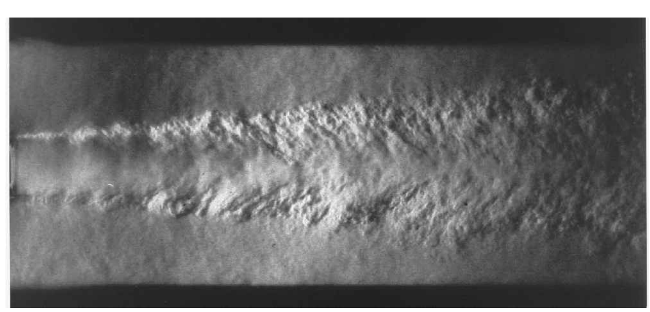


**Fig. 2 Ducted rectangular jet focused schlieren.**

**Table 1 Convective Mach numbers of two-dimensional shear layers and rectangular jets**

| Primary nozzle pressure ratio | Secondary nozzle pressure ratio | Primary total temperature, K | Secondary total temperature, K | $M_C$ P&R | $M_C$ Splitter | $M_C$ Rectangular jet | $M_C$ Ducted rectangular jet |
|---|---|---|---|---|---|---|---|
| 3.543 | 1.01 | 617 | 322 | 0.777 | 0.78 | 0.87 | 0.865 |
| 3.517 | 1.01 | 305 | 299 | 0.584 | 0.58 | 0.80 | 0.794 |
| 3.510 | 1.10 | 931 | 330 | 0.547 | 0.55 | 0.76 | 0.756 |

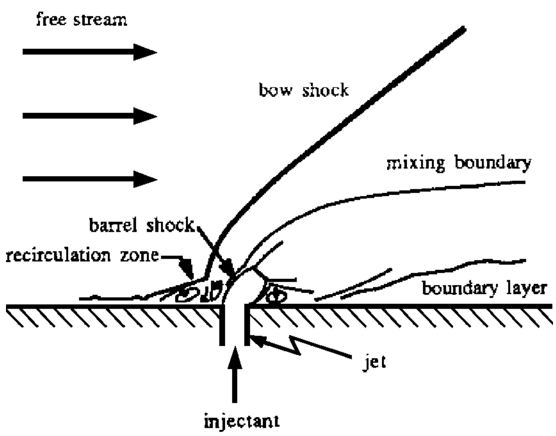



**Fig. 3 Normal fuel injection (from Ref. 16).**

applicable. We shall come back to this issue when we explore the use of counterflow technology to enhance mixing in supersonic streams.

To summarize, it appears that theoretical, numerical, and experimental studies point to a reduction in the $\langle p'u' \rangle$ correlation to transfer energy to spanwise $v'$ fluctuations and normalized kinematic Reynolds stress $u'v'/\sqrt{u'^2}\sqrt{v'^2}$. This reduction leads to a strong anisotropic swept turbulent structure in the shear layer that is predicted by linear instability wave theory through observed reduction in amplitude growth rates. We have also seen that the rectangular jet large-scale structure convects at a different rate than that associated with the two-dimensional splitter flowfield. Evidently the large-scale structures associated with each shear layer mutually interact by virtue of their induced hydrodynamic pressure field to convect turbulence structure at a different phase speed.

## Reacting Compressible Free Shear Layer Mixing

The preceding exposition describes how turbulent mixing efficiency decreases with increasing convective Mach number. The growth rate of reacting shear layers is expected to be modified by the heat released through combustion. Heat release lowers mixed density, which gives rise to a favorable streamwise pressure gradient as discovered by Hermanson and Dimotakis.[13] They evaluated the planar turbulent shear layer with large heat release from the reaction of a mixture of hydrogen and fluorine with air. Their study was conducted at very low speed in the order of 20 m/s. What they discovered was that turbulent mixing efficiency was reduced at a given convective Mach number through observed reduction in the shear layer growth rate parameter $C_\delta$. They argued that the outward displacement velocity due to heat release interferes with the mechanism of entrainment. Thus, despite the large increase in shear layer spanwise displacement, the efficiency of mixing of an oxidizer with fuel diminished due to the large change in density and streamwise acceleration of the flow.

The experimental results obtained by Hermanson and Dimotakis[13] were later confirmed by Menon and Fernando,[14] who conducted a direct numerical simulation (DNS) of the low Reynolds number reacting planar shear layer. The DNS calculation only considered chemical heat release from a binary irreversible reaction. The DNS showed that mixing efficiency and the amount of product formation decreased with convective Mach number for subsonic flow. At supersonic speeds, the DNS predicted the appearance of strong streamwise vortices, as in the nonreacting shear layer, but heat release produced increased shear layer growth due to an increase in the associated dilatational field.

Drummond[15] also performed simulations of the reacting and nonreacting supersonic planar shear layer using spectral methods for the Navier–Stokes solver. His results showed diminished mixing with chemical heat release at supersonic speeds in contrast to the results of Menon and Fernando.[14]

**Table 2 List of passive and active mixing enhancement concepts**

| Mixing device | Physical mechanism | Cited references |
|---|---|---|
| | *Passive* | |
| Ramps | Streamwise vorticity | 16–23 |
| Tabs | Streamwise vorticity | 24–28 |
| Lobe mixers | Streamwise vorticity | 29–30 |
| Chevrons | Streamwise vorticity | |
| Vanes | Swirl | 31–37 |
| Port geometry | Excitation of swept structures | 38–42 |
| Transverse injection | Transverse curvature | 43–45 |
| Shock/shear layer interaction | Excitation of large scales | 46–49 |
| Counterflow | Self-excited resonance | 50–51 |
| Backward facing step | Self-excited resonance | 52–53 |
| Cavities | Acoustic excitation | 54 |
| | *Active* | |
| Vibrating splitter/wire | Forced excitation of large scale | 55 |
| Pulsed jet | Forced excitation of large scale | 56 |
| Helmholtz resonators | Forced excitation of large scale | 57–61 |
| Piezoelectric actuators | Forced excitation of large scale | 62–63 |
| Acoustic excitation | Forced excitation of large scale | 64–69 |
| Wavy wall | Spatial excitation of large scale | |
| Flip-flop nozzle | Increased transverse curvature | 70 |

## Mixing Enhancement Concepts

The preceding section presents an outline of some features that control the compressible mixing associated with dissimilar gases both with and without chemical reaction. A number of concepts have been applied in the past to increase scramjet combustion efficiency through controlled forced mixing. The scramjet has an additional requirement associated with the fuel residence time requirement to heat fuel to enable combustion. Table 2 provides a list of concepts that have previously been applied to the scramjet combustor and jet noise reduction programs. A brief summary of those efforts follows.

## Passive Mixing Devices

The earliest scramjet combustor designs involved normal injection of fuel into the supersonic airstream as shown in Fig. 3 from the work of Lee et al.[16] As illustrated, the injectant produces a detached normal shock upstream of the jet giving rise to separated air zones both upstream and downstream of the jet. This translates into significant losses in total pressure and, consequently, scramjet cycle efficiency. However, combustion can be achieved in very short distances from injection because the separation zone acts a flame holder. Studies with parallel injection minimize cycle performance losses, but have extremely low combustion efficiency due to poor mixing at supersonic speed as discussed earlier. From studies such as these, it was obvious that a better solution was needed. Dimotakis[17] concluded that parallel mixing could work if assisted

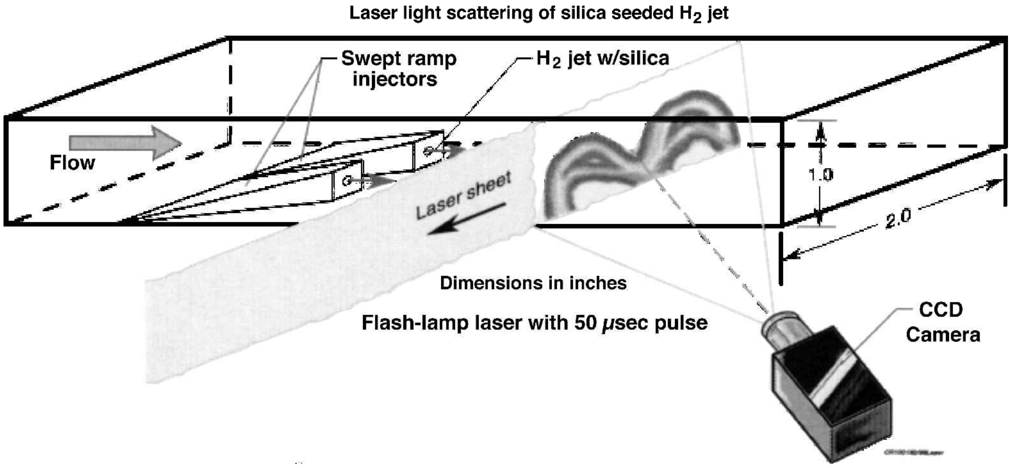



Fig. 4 Generation of counter-rotating streamwise vorticity by ramp fuel injector (from Ref. 18).

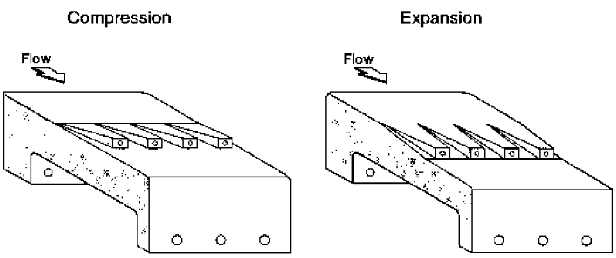
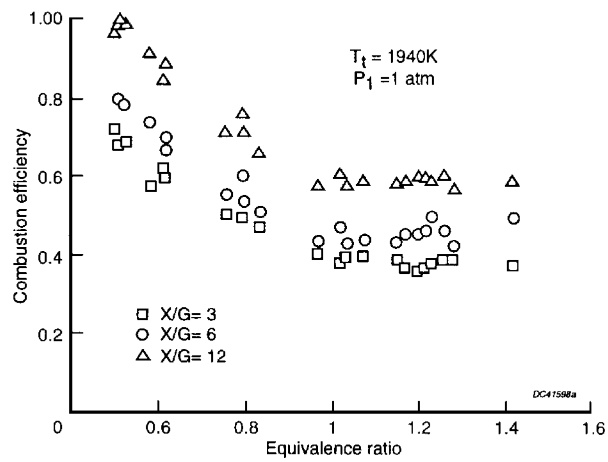
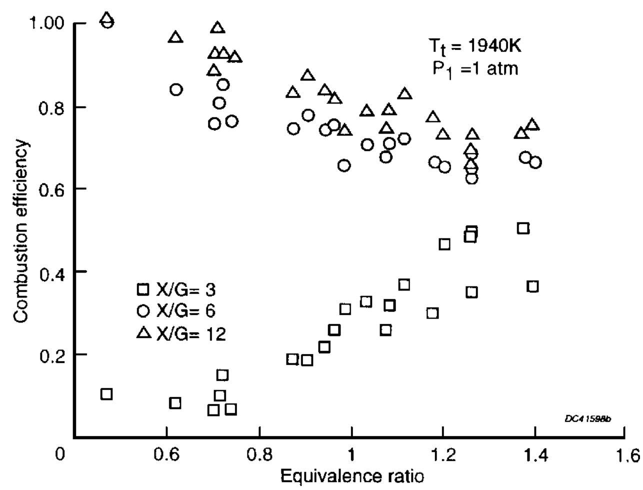



**Fig. 5 Scramjet fuel injector compression and expansion ramps (from Ref. 19).**




a) Compression ramp




b) Expansion ramp

**Fig. 6 Combustion efficiency of compression and expansion ramp fuel injectors (from Ref. 19).**

with generation of axial vorticity. This gave rise to new studies involving near parallel injection from ramps, which can generate axial vorticity.

### Ramp Fuel Injectors

The flow over a ramp induces a pair of counter-rotating streamwise vortices as shown in Fig. 4 (from the work of Rogers et al.[18]). Supersonic flow over a ramp generates both shock and expansion waves. This leads to the generation of the vorticity by baroclinic torque. Baroclinic torque is exerted on a fluid flow whenever the density and pressure gradients are not parallel as is evident in the transport equation for vorticity as given by

$$\rho\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\bar{\omega}}{\rho}\right) = \frac{1}{\rho}\nabla\rho \times \nabla p \tag{6}$$

Numerous ramp designs have been investigated. For example, in the work of Stouffer et al.,[19] an array of compression and expansion style ramps were investigated whose geometry is shown in Fig. 5. In these studies, hydrogen was used as the fuel and air as the oxidizer. In these concepts, rapid turns in geometry induce formation of shocks, which for compression ramps occur at the base and for expansion ramps at the trough. Dramatic differences were found in mixing and combustion efficiency between these two styles. It is well known that, for the same ramp angle, vorticity shed from a compression ramp induces much stronger vorticity than that from an expansion cycle due to differences in projected stream area. Despite significantly improved fuel/air mixing from the compression ramps, the expansion ramps achieved higher combustor efficiency, as shown in Figs. 6a and 6b, respectively. This result may be expected because good combustion efficiency is achieved with mixing at small scales. The extremely strong vorticity of the compression style ramp, while increasing fuel/air contact area also provides a centrifuge that diminishes opportunity for mixing.

The problem with generation of strong axial vorticity is that it may remain stable suppressing the growth of transverse scales used for mixing beyond the region required for combustion. For combustion choked scramjet designs, this would be disastrous. Leibovich[20] has developed a criterion that permits one to select ramp angles that shed unstable vorticity. His criterion to generate a stable vortex requires that

$$V\frac{\mathrm{d}\Omega}{\mathrm{d}r}\left[\frac{\mathrm{d}\Omega}{\mathrm{d}r}\frac{\mathrm{d}\Gamma}{\mathrm{d}r} + \left(\frac{\mathrm{d}U}{\mathrm{d}r}\right)^2\right] < 0 \tag{7}$$

where $V$ is the vortex swirl velocity, $\Gamma$ the vortex circulation, and $\Omega$ the vortex angular velocity. If Eq. (7) is satisfied, then the shed vortex is unstable to small disturbances. Circulation from a ramp can be calculated from the relation $\Gamma = 2Uh\tan(\alpha)$, where $h$ is the ramp height and $\alpha$ is the ramp angle. Using the Leibovich criterion,[20] unstable vortices are produced for ramp angles when

$$\alpha \leq \tan^{-1}\left[\frac{\pi r(\mathrm{d}U/\mathrm{d}r)}{2h(U/r - \mathrm{d}U/\mathrm{d}r)}\right]^{\frac{1}{2}} \tag{8}$$

In research related to the reduction of supersonic jet noise, the mixing of hot supersonic air with high subsonic cold air led to the development of the lobed mixer. The general concept for lobed mixers for noise reduction has been studied in Ref. 9 using a lobe concept for scramjets. In this concept, each stream is diverted into opposite directions. The mean flow shear at the stream interface generates the streamwise vortex. Good lobe geometry design requires no flow separation to induce vorticity, and excellent aeroperformance is achieved. The vortical lobe style utilizes squared-off ramp style geometry, like those of the compression style scramjet ramps of Stouffer et al.[19] The axial lobe mixer utilizes smooth gradual geometry that reduces the mean shear between fluid streams relative to that of the vortical mixer. The vortical mixer has a much higher mixing efficiency in the vicinity of the exit of the lobes, but eventually the axial mixer achieves a higher mixing efficiency $\eta$ at some distance from the lobe trailing edge. Here,

$$\eta = 1 - \iint \frac{(T_{0X} - T_{0M})}{(T_{0I - T_{0M}})}\,\mathrm{d}y\,\mathrm{d}z, \qquad T_{0M} = \frac{W_1 T_1 + W_2 T_2}{W_1 + W_2} \tag{9}$$

where $T_{0X}$, $T_{0I}$, and $T_{0M}$ are the total temperatures at some downstream axial station, lobe trailing edge, and fully mixed temperature, respectively. $W_1$ and $W_2$ are the individual stream mass flux. We see that, even in the case of noise reduction, that initial streamwise vortex strength plays an important role in deciding overall system effectiveness.

Ramp fuel injectors have also been designed with sweep to control shock and expansion waves. Tests conducted at NASA Langley Research Center show that combustion efficiency of swept ramp injectors is much improved compared to those without sweep, as shown in Fig. 7 (from Ref. 21). Figure 7 also contains curves of the best that can be expected between parallel and normal injection. As can be seen, the ramps with sweep approach that achieved with normal injection. The effect of ramp sweep was numerically investigated by Drummond et al.[22] and Donohue et al.,[23] who also performed experimental planar laser induced iodine fluorescence measurements. Their calculations confirm the importance of using sweep for the ramp injectors. The comparison in Fig. 8, however, shows that the calculated strength of streamwise vorticity is less than observed experimentally, presumably due to insufficient grid resolution.

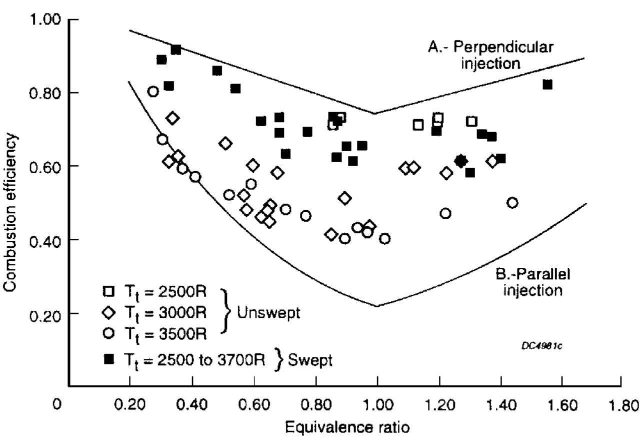



**Fig. 7 Combustion efficiency of swept and unswept fuel injector ramps (from Ref. 21).**

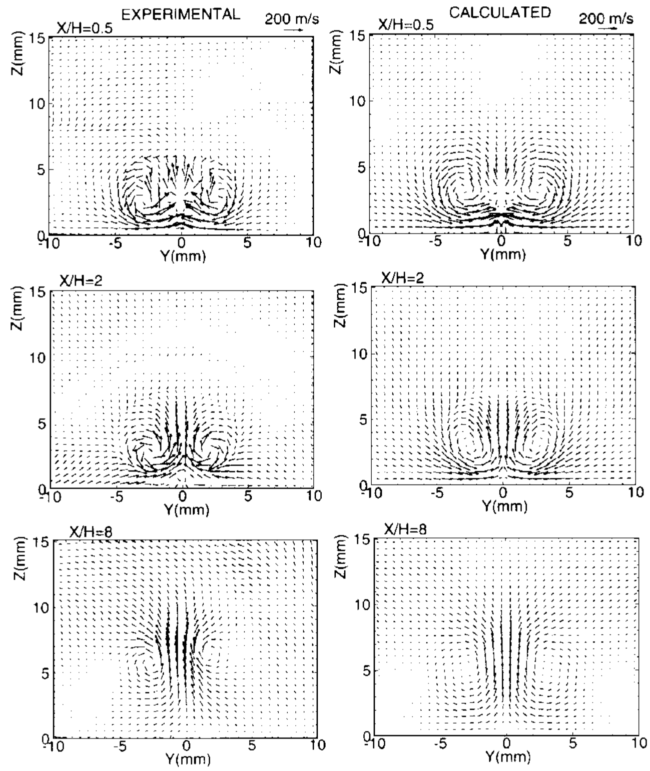



**Fig. 8 Measured and predicted crossflow velocities (from Ref. 23).**

## Tabs

Another way to inject streamwise vorticity to enhance mixing is through the use of prism-shaped devices known as tabs. These devices have been used in studies aimed at reducing supersonic jet noise through enhanced mixing. Two distinctly different concepts have been employed with the use of tabs. The first concept explicitly attempts to introduce streamwise vorticity to increase area contact between low- and high-speed streams. The second concept introduces streamwise vorticity explicitly to stimulate large-scale shear layer instabilities through injection of additional shear layer velocity inflection profiles. The design of the tab element is significantly different for each concept. In the first concept, the tabs are large and spaced far apart. In the second concept, the tabs are of the order of the initial shear layer momentum thickness and spaced very tightly to generate a continuous inflection sheet in the initial shear layer.

Seiner and Grosch[24] investigated experimentally and numerically the application of tabs associated with the first concept to a round axisymmetric nozzle. The tabs are inclined away from the nozzle lip at a ramp angle of 45 deg. Supersonic flow over these tabs clearly leads to separated flow and the generation of counter-rotating vorticity like that obtained from the ramp style fuel injectors. The total projected blockage area of the tabs was 3% of the nozzle exit area. To determine the effectiveness of these tab adaptations to the nozzle lip, measurements were conducted to assess mass flow entrainment of coflowing air into the jet stream. Figure 9 shows, for several tab adaptations, the mass flux past a given axial station normalized by the mass flux at the nozzle exit. One of the tab designs, the fence tab, was taken directly from the work of Ahuja and Brown.[25] As can be seen, by 15 jet diameters downstream, the jet stream contains almost 2.5 times the original mass flux, even for the baseline round nozzle without tabs. This is similar to the result obtained by Zaman et al.[26] When these results are normalized by the mass flux of the baseline nozzle at a given crossplane, one finds, as shown in Fig. 10, that some of the tab configurations appear to continue to entrain fluid far from the nozzle exit, but others actually can shut down entrainment far from the nozzle exit. Thus, in addition to the Leibovich criterion[20] on vortex stability, there must be yet some other mixing parameter related to the spacing of injected vorticity because the larger number of tabs per nozzle tend to produce

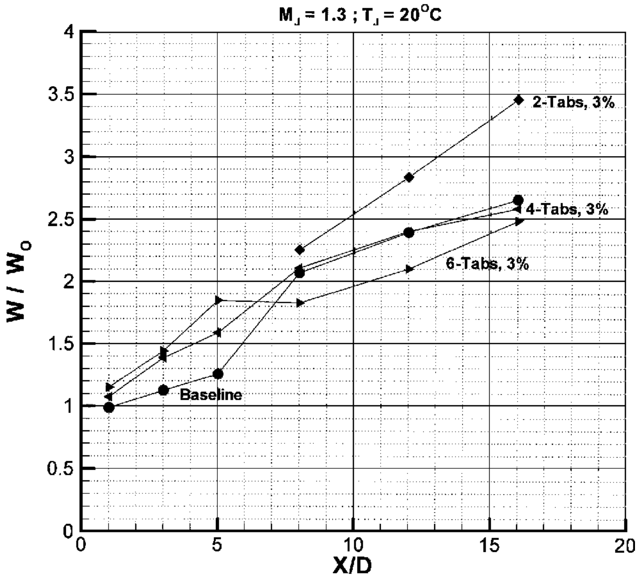



**Fig. 9 Measured mass entrainment of various tabbed nozzles relative to nozzle mass flow (from Ref. 24).**

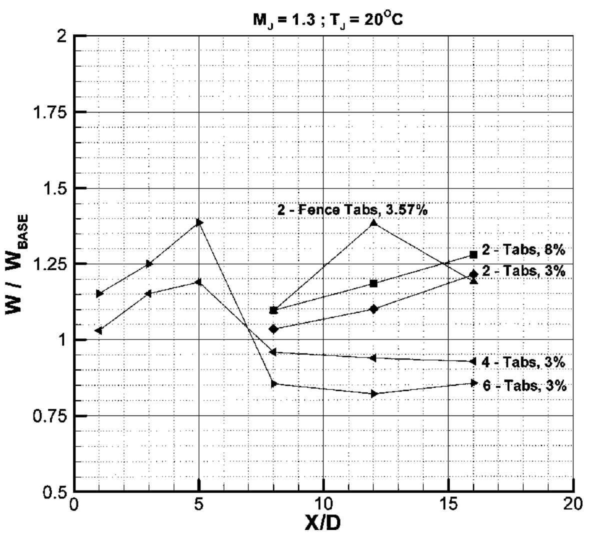



**Fig. 10 Measured mass entrainment of various tabbed nozzles relative to baseline nozzle mass flow entrainment (from Ref. 24).**

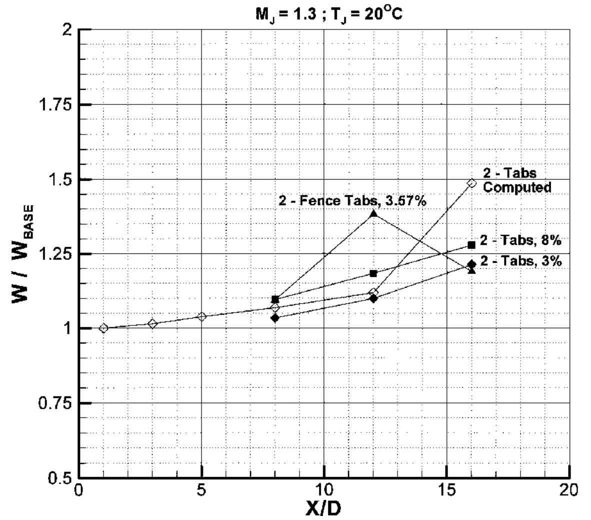



**Fig. 11 Predicted mass entrainment of various tabbed nozzles relative to baseline nozzle mass flow entrainment (from Ref. 24).**

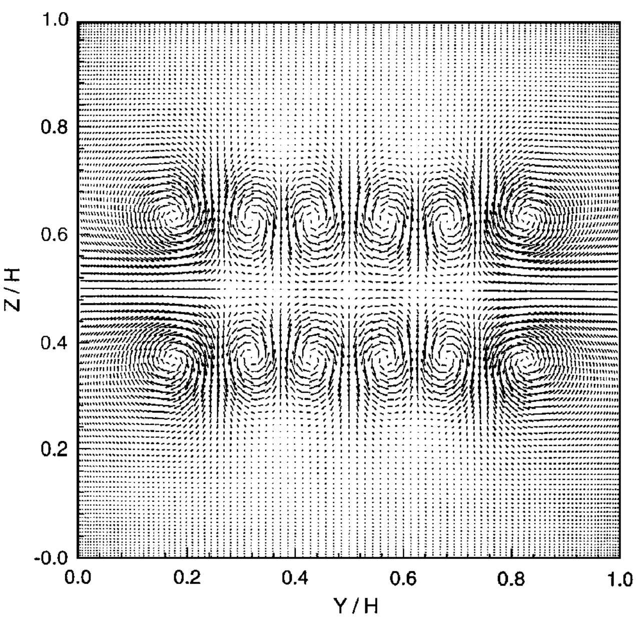



**Fig. 12 Numerical simulation of flow from a slot nozzle with tab array and coflow (from Ref. 28).**

extremely stable vortex orbits and impede mixing. Figure 11 shows that Navier–Stokes simulations faithfully capture the essence of the entrainment process by virtue of agreement between measured and predicted entrainment.

In application of the second concept, the tab size, ramp angle, and spacing is determined from solution of the compressible Rayleigh equation and solution of Navier–Stokes in the near field to set parameters in the Rayleigh equation. The Rayleigh equation is an instability model for large-scale turbulence structure. Based on Brown and Roshko's[2] results, the growth of these large scales is related to shear layer mixing effectiveness. The wavelike nature for assumed instability waves in cylindrical geometry can be represented, for example, for pressure as

$$p(r, \theta, z, t) = A(x)\hat{p}(r)\exp[i(\alpha x + n\theta - \omega t)] \qquad (10)$$

where the streamwise wave number is $\alpha = \alpha_r + i_{\alpha i}$ and the phase speed is $c = \omega/\alpha_r$. When this form is substituted into the compressible Rayleigh equation, one can determine the overall wave amplitude growth from

$$A(x) = A_0(x_0)\exp\left\{\int_{x_0}^{x}(i\alpha_r - \alpha_i)\,\mathrm{d}x\right\} \qquad (11)$$

The growth is controlled by $\alpha_i$, which in turn is controlled by the strength and size of the shear layer inflection characterized by $1/(\mathrm{d}^2U/\mathrm{d}r^2)$. This concept was computationally applied by Berkooz et al.[27] to a two-dimensional shear with an array of tabs at the trailing edge of a two stream splitter. Their results showed enhanced growth rates for the most unstable shear layer mode and substantial increase in mixing efficiency.

Grosch et al.[28] also utilized a tab design based on the Rayleigh equation and applied it to the ducted supersonic slot jet with heated flow and subsonic coflow. Figure 12 shows the resulting crossplane velocity vectors when three tabs were applied to each stream splitter. Figure 13 shows the mixing enhancement in terms of the earlier defined mixing parameter $\eta$. As can be seen, in this case, a substantial increase in mixing efficiency occurs with the larger number of tabs where initial vortex strength per tab is adjusted so that the summed input is equal for all cases. Presumably, the high-density array may have excited a near continuous velocity defect to obtain maximum growth rate of large-scale structure. In this tab adaptation, the prism shape size is very small relative to adaptations with the first concept. Performance losses are expected to be small.

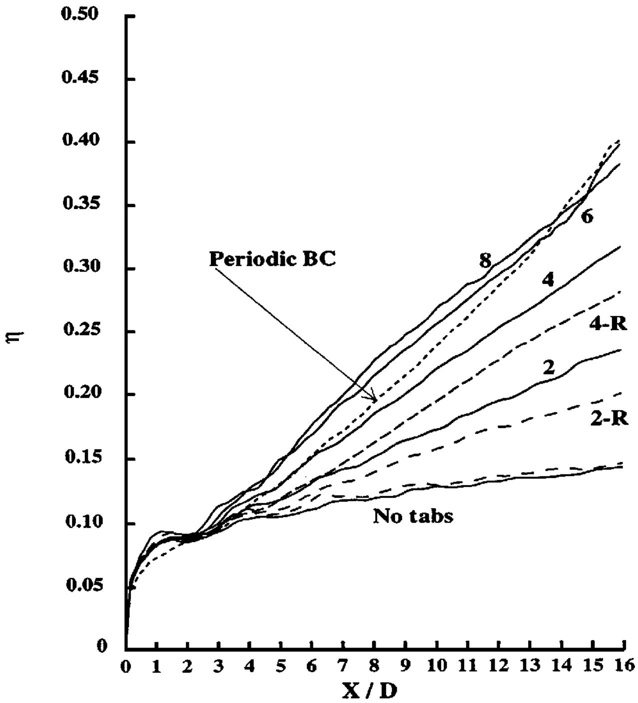



Fig. 13 Comparison of mixing effectiveness for various tab arrays (from Ref. 28).

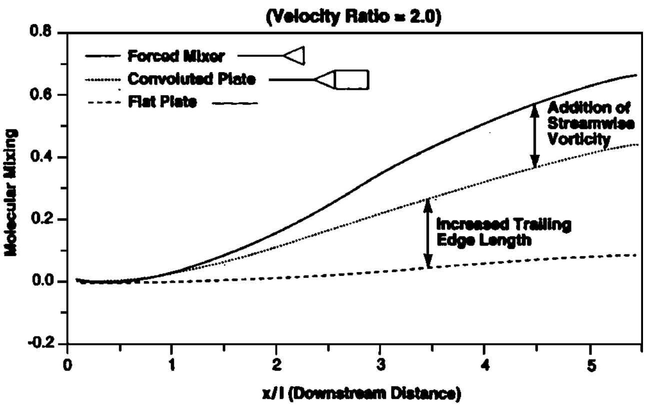



Fig. 14 Comparison of mixing efficiency between lobe mixers (from Ref. 30).

We see that tablike structures can be used to enhance shear layer mixing efficiency. It is conceivable that the ramp style fuel injector ports could also contain small tablike structures at their exit to enhance mixing. Note that tab geometry shrinks with increasing Mach number so that they may be well suited for this application.

### Lobe Mixers

Marble et al.[29] have studied the lobe mixer for scramjet applications. Fuel is injected through the lobes at a small angle to the base wall. The air is ducted between the lobes and goes through an expansion wave as it is directed with a small angle to the base wall. Streamwise vorticity is generated along the entire periphery of the exhaust lip where fuel and air interface. This method leads to excellent mixing efficiency but suffers high aerodynamic losses due to the squared-off geometry. Also there is no recirculation zone for flame holding with this concept. Tew et al.[30] more recently studied a modification to this design. Figure 14 shows that the forced mixer of Marble et al.[29] achieves the highest mixing efficiency, but that the convoluted plate is still substantially superior to the mixing layer without injected streamwise vorticity. The aeroperformance losses were substantially lower than that associated with the mixer of Marble et al.

### Chevrons

Chevrons have wide application for vehicles that are designed for low radar cross section. However, NASA has also used them in programs for noise reduction for subsonic and supersonic exhaust nozzles. They have been combined with lobes for both the core and fan streams of separate flow nozzles. These devices are an effective means to enhance mixing efficiency because they produce streamwise vorticity when angled into the high-speed flow, increase perimeter mixing, and have very high aerodynamic performance.

### Swirl

From our earlier discussion, mixing efficiency improves if one can directly excite lateral perturbations into the shear layer. Naughton and Settles[31] and Naughton et al.[32] examined the effect of swirl on mixing efficiency for the nonreacting supersonic shear layer. In their experiment, helium at Mach 4 was injected into a coflowing supersonic stream whose Mach number varied from 2.8 to 4.5. The plenum of the helium jet supply was equipped with varies that could impart flow angles of 0, 30, 45, and 60 deg. Detailed measurements indicated that swirl numbers ranging from 0 to 0.106 could be applied to the helium jet. The swirl number is the ratio of the axial components of angular to linear momentum flux. It represents a ratio of the angular torque to axial thrust. Their planar laser scattering results indicated approximately a 40% increase in plume spread rate at the higher values of swirl number.

The effect of swirl on mixing has also been studied by Kraus,[33] who injected helium at a 30-deg flow angle into a supersonic airstream. Kraus also found improvement in shear layer mixing with swirl, but found penetration from the injector port to be the same with and without swirl.

More recently Jacobsen et al.[34] studied mixing improvements associated with an array of fuel ports with normal injection. The helium flow from the injectors was also given an angular component of velocity. They achieved not only improved mixing efficiency, but also observed lower total pressure losses.

Theoretical studies on swirling jets have been conducted by Martin and Meiberg[35] using linear stability. They found exponentially growing helical waves under swirl perturbation, but this may be expected because they assumed inviscid flow. Also of importance is the research conducted by Farokhi et al.[36] on the use of swirl to enhance mixing for jet noise reduction and the research of Dutton,[37] who developed developed a computational procedure to design supersonic nozzles containing swirl and achieving high aerodynamic performance.

### Port Geometry

The ramp fuel injector geometries already discussed utilize round ports. However, it is well known that nozzle geometry by itself can be used to excite transverse scales used to enhance mixing. It is also known that supersonic flow from nozzles located close together can achieve self-excited resonance and achieve very high mixing efficiency. Figure 15 shows a comparison between round

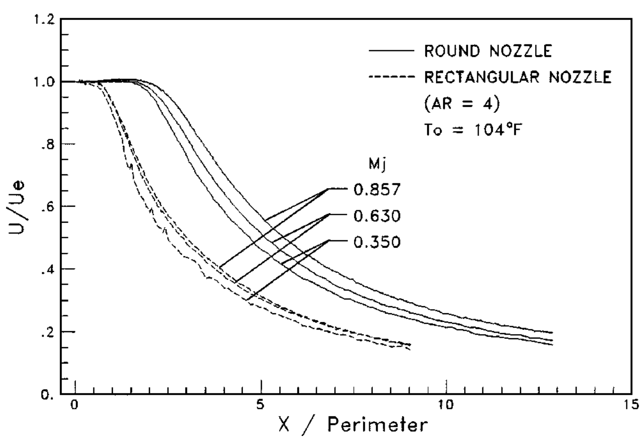



Fig. 15 Jet centerline decay of round and rectangular nozzles.

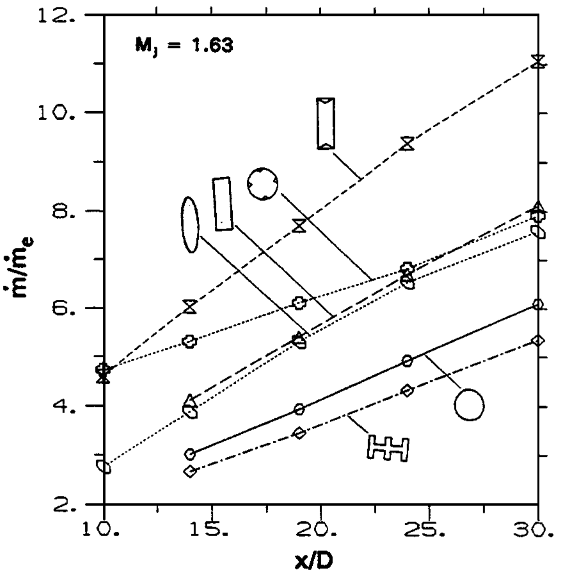



**Fig. 16 Mass flow entrainment of several nozzle geometries (from Ref. 38).**

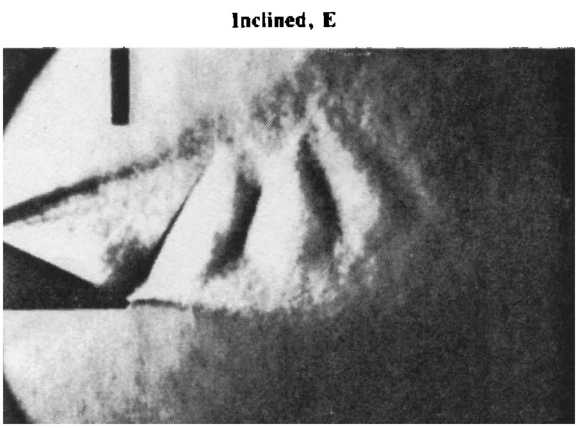



**Fig. 17 Enhanced spreading and vectoring of intermediate origin nozzle (from Ref. 42).**

and rectangular jet plume centerline profiles. As can be seen, the rectangular jet has a much higher decay rate indicating much better mixing with surrounding coflow than the round nozzle. Figure 16, from Zaman,[38] shows for several nozzle geometries, including elliptic, that mass flow entrainment can be significantly augmented through selection of nonround geometry. Of the nozzles shown in Fig. 16, the nozzle that achieves the highest entrainment rate, relative to the nozzle's initial mass flux, is the tabbed rectangular nozzle. Zaman's research was conducted with a jet nozzle exit Mach number of 1.63.

It is well understood from the early studies of Ho and Gutmark[39] that elliptic nozzle geometry can entrain mass several times higher than its equivalent round counterpart. The nonaxisymmetric shear layer vortex structure induces streamwise vorticity needed for mixing. Schadow et al.[40] studied mixing enhancement in reacting round and elliptic nozzles. They found substantial improvement in mixing with the elliptical nozzles for both reacting and nonreacting jets. Numerical studies conducted by Mutter et al.[41] show good comparison for the nonreacting elliptic nozzle study of Ho and Gutmark[39]; however, the calculations showed diminished mixing efficiency with the reacting jet flow.

Wlezien and Kibens[42] have studied nozzle geometry with intermediate origins. An example phased, averaged schlieren record for their inclined E nozzle is shown in Fig. 17. As can be seen, the flow from this nozzle is vectored and spreading at a high rate for what is customarily expected for a round nozzle. Both of these features are important for fuel port geometry. This research suggests that the fuel port geometry for a scramjet could be more effective if it contained asymmetric features.

### Streamline Curvature

Bradshaw[43] studied the effect of streamline curvature on a shear flow where he found that a flow remains stable if the radial gradient of angular momentum is positive, and it is unstable if the radial gradient is negative. Thus, in the flow over a curved wall, the shear layer toward the inner wall has reduced shear growth, and the shear layer away from the wall has augmented growth. Kibens[44] studied this effect and achieved substantial shear layer growth when the radial gradient of angular momentum was positive. These effects are present in those studies where fuel is normally injected into an airstream. Destabilization of the shear layer for this type of method for fuel injection could be examined from the stability associated with streamline curvature.

### Shock/Shear Layer Interaction

Ribner[45] was among the first to demonstrate from linear theory turbulence amplification on interaction with a plane shock wave. Menon[46] experimentally studied shock wave induced mixing enhancement for scramjet combustors. The experiment was conducted with nitrogen as the oxidizer and helium as the fuel. The helium was injected parallel over a rearward-facing step that would act as a flame holder. A small wedge was used to generate an oblique shock. His Rayleigh scattering concentration measurements showed a substantial increase in mixing with shock interaction. In the numerical simulations of Nixon et al.,[47] large increases in the turbulence correlation involving $\langle p'u' \rangle$ were observed when a shock wave interacts with a shear layer. This is a term that drives spanwise scales used in mixing. Drummond[48] numerically considered shock/shear layer interaction for a reacting shear layer and found that both mixing and combustion efficiency improved over the standard ramp fuel injector discussed earlier.

### Counterflow

This concept utilizes the theory of Huerre and Monkewitz[12] for self-excited resonance due to acoustic feedback. The theory is based on the absolute or temporal instabilities associated with solution of the Rayleigh equation.

Strykowski and Niccum[49] studied the absolute instability of mixing layers using an apparatus like that shown in Fig. 18. Here, the primary nozzle flow is surrounded with a collar, which enables acoustic disturbances from the primary flow to be drawn back to the lip of the primary nozzle and excite a feedback resonant loop. This is similar to the process associated with screech tones in supersonic shock containing jets. Using this method, Strykowski et al.[50] demonstrated that large increases in mixing efficiency could be achieved with suction backflow only a few percent of the primary flow. Figure 18 shows the dramatic effect on mixing. Figure 19a shows the baseline nozzle and Fig. 19b with counterflow on.

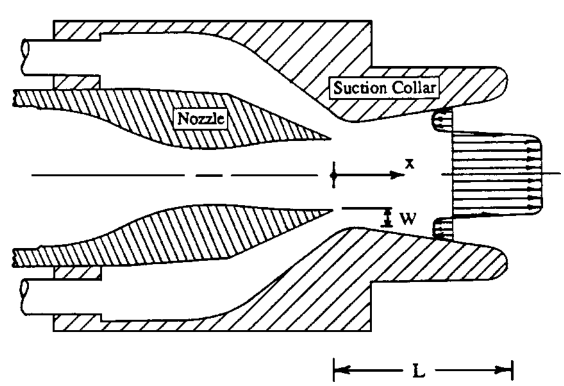
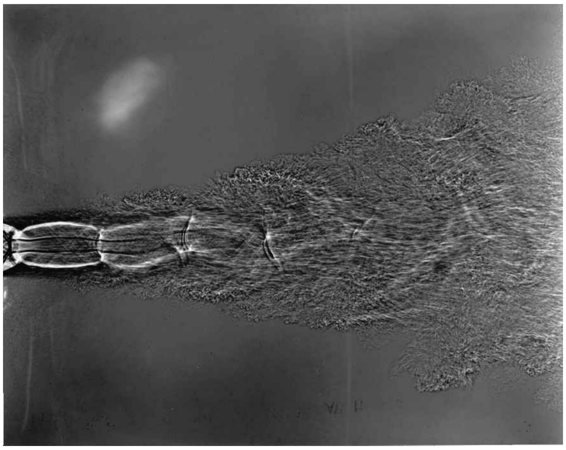
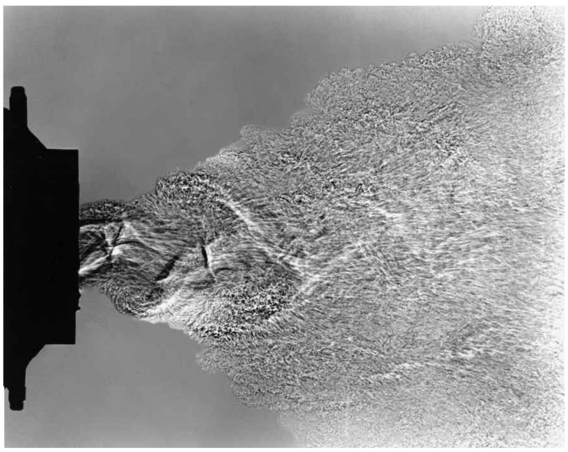



**Fig. 18 Schematic of counterflow concept (from Ref. 49).**



**a) Counterflow off**



**b) Counterflow on**

**Fig. 19 Mixing enhancement with counterflow at $M_j = 1.45$ (from Ref. 50).**

This concept would provide outstanding flame-holding capability if applied to a scramjet. Also by providing nonuniform suction, the jet flow can be vectored over substantial angles.

### Backward-Facing and Multistep Nozzle

This concept also involves self-excited resonance, but is natural as compared to forced in the counterflow studies. Fernando and Menon[51] have conducted experiments in the backward facing step. The purpose of the backward facing step is to provide flame-holding capability for parallel injection and perhaps excite acoustic resonance behavior. Their experiments demonstrated a twofold increase on mixing of the lower shear layer facing the lower wall. Their schlieren records and the lack of increased mixing of the outer shear layer suggest that acoustic resonance was not activated in their experiments.

Another variation of the use of the backward-facing step is associated with the multistep nozzle experiments of Gutmark et al.[52] In their experiments, they found that the multistep nozzle introduced small scale turbulence into the flow, which led to substantially improved combustion.

### Cavities

Appropriately designed, a cavity can be an effective tool to achieve self-excited resonance due to acoustic excitation of a free shear layer. Recently Yu et al.[53] investigated several cavities as a means of improving scramjet combustion efficiency. Their apparatus is shown in Fig. 20. This apparatus permitted the study to be done using a reacting shear layer. Figure 21 also shows schlieren records associated with several of their designs. They found that the most efficient configuration was the two-step cavity with an inclined

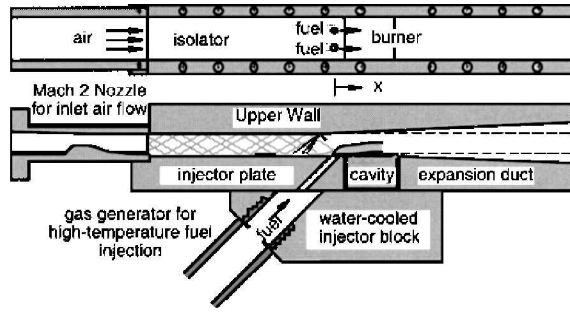



**Fig. 20 Acoustic cavity resonance concept for scramjet (from Ref. 53).**

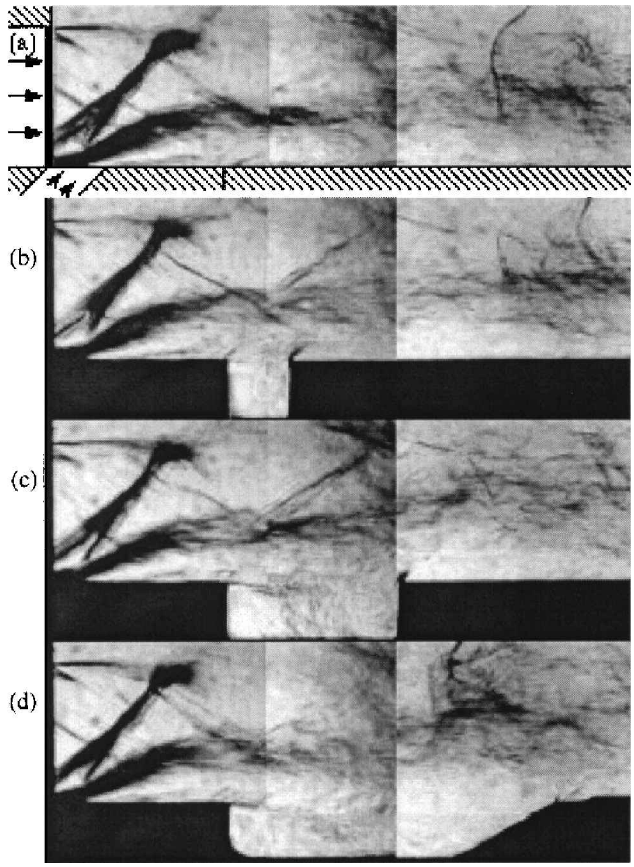



**Fig. 21 Cavity design and schlieren results (from Ref. 53).**

wall. This design led to the highest increase in combustor pressure, and exit recovery temperature suggested significant improvement in combustion efficiency.

## Active Mixing Devices

The utilization of devices that are actively controlled to enhance mixing efficiency for scramjet applications appears remote when one considers the combustor environment. However, understanding time-dependent behavior of the reacting mixing layer is paramount to development of an optimized combustor design. We will look at but a few of the devices that have been studied as a means for controlling jet noise emission.

### Vibrating Splitter/Wire

It is well known that a wire or thin plate when placed in a stream will shed vorticity whose principle frequency in terms of Strouhal number $Sr$, is given by $Sr = fL/U \approx 0.2$, where $L$ is the characteristic dimension of the wire or thin plate. One can then use this to design a wire or plate that is extended across the shear layer. Through tensioning, the pressure fluctuations in the shed wake can

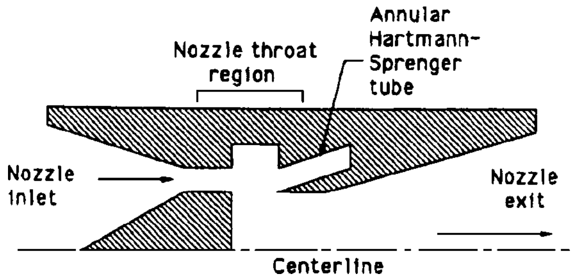



**Fig. 22 Hydrogen fuel injector nozzle using acoustic pulsing from Hartmann–Sprenger tube (from Ref. 55).**

have controlled input into the shear layer to excite the fastest growing instability wave. Vandsburger and Ding[54] studied the effects of a wire stretched across a jet shear layer and found they could double the mixing efficiency of the shear layer. Newer methods using wires now consider deployment of a wire down the center of jet flow into the region of the potential core. Here the free end is excited to shed vorticity and produce an accelerated transfer of energy from large to small turbulence scales. This would be good for enhancing combustion efficiency.

### Pulsed Jet

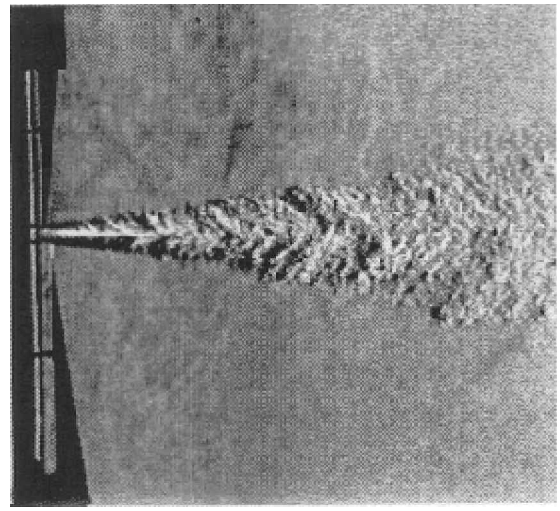
As will become clear from the subsequent discussion on acoustic excitation, pulsing a jet at the columnar instability frequency leads to enhanced mixing efficiency. Recently Bogdanoff[55] described a robust method for pulsing the fuel injector ports in a scramjet combustor. Figure 22 shows a schematic of his proposed scheme. The cavity serves to generate strong acoustic waves, which couple to the modes of an annular Hartmann–Sprenger tube that is excited by shed frequencies from the sharp lip. This device is capable of developing pressure oscillations on the order of the fuel injector pressure. The device uses no moving parts, can withstand temperatures within the combustor, and can be tuned to deliver frequencies that drive shear layer instability waves. The device tuning can be actively controlled by variation of the geometry of the cavity and annular tube.

### Helmholtz Resonators

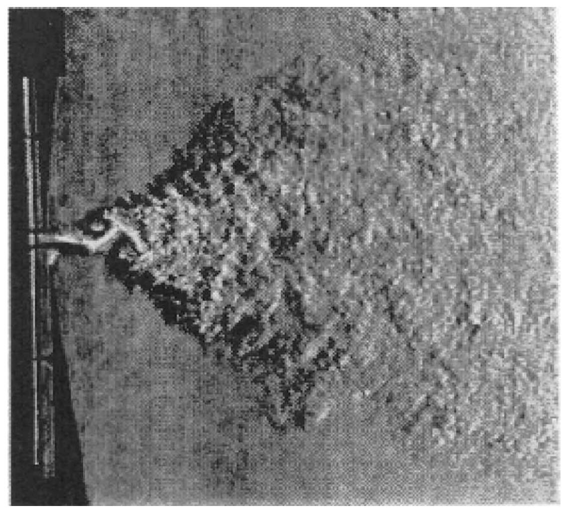
Helmholtz resonators were in use in the last century as a means for canceling low-frequency noise in ducts. Panton[56] and Chanaud[57] have more recently investigated the effects of geometry on resonance frequency of the Helmholtz cavity. Two recent adaptations of the Helmholtz cavity have been studied that permit the use of these devices as a phase-controlled force input to a fluid.

The first is one designed by Corke et al.[58] for jet noise control is known as a glow discharge device. It is a Helmholtz cavity where beween the cathode and anode in the interior cavity a plasma is created to operate in the Townsend discharge range. In this range, a pure oscillating electrical signal can be conducted through the plasma. This produces an oscillating internal temperature field that produces at cavity resonance a strong efflux of velocity from the orifice. Hence, the force input is required for excitation of a shear layer. The second adaptation is that due to Amitay et al.,[59] who utilized the NASA thunder device at the bottom of the Helmholtz cavity. Because this is a Helmholtz cavity, the total mass flux over time is zero.

Neither one of the devices has thus far been able to demonstrate sufficient force to excite a supersonic shear layer to enhance mixing efficiency. However, little is known on how to optimize the design of these devices. Toward this direction, Cain et al.[60] have numerically simulated the flow from the thunder device and obtained reasonable agreement with the experiments of Wiltze and Glezer.[61] However, their simulation did not include the rich internal acoustic field of the cavity that would be required to improve physical modeling of the device.



**a) Flap off**



**b) Flap on**

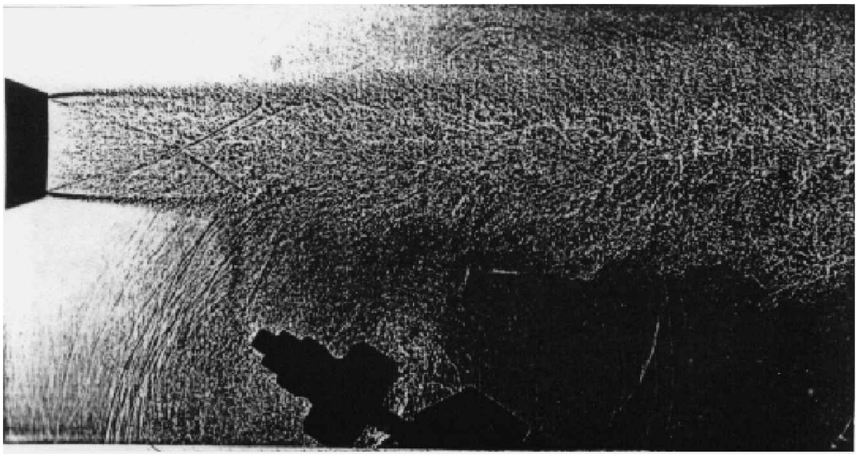
**Fig. 23 Mixing enhancement with piezoelectric flap (from V. Kibens, private communication, 1993).**

### Piezoelectric Actuators

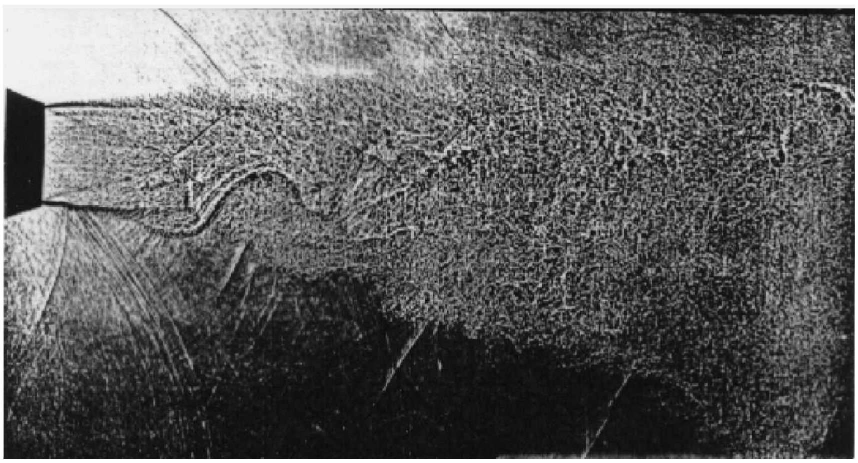
Wiltze and Glezer[61] have examined the effect of oscillating flaps located at the trailing edge of a nozzle. These flaps were driven to resonance by a piezoelectric actuator at a frequency that the free shear layer could support for growth of large-scale structure. Figure 23, from the work on Kibens (private communication, 1993), who conducted similar experiments, shows the very large increase obtained when the flaps were oscillating. The very exciting result from the Wiltze and Glezer[61] study was that their measurements indicated a substantial increase in small-scale turbulence structure. This would be beneficial for improving combustion efficiency for scramjets.

### Acoustic Excitation

Acoustic excitation of jets has been of concern to the aircraft noise community for some time. Combustion instabilities produce strong acoustic tones in aircraft engines, and it was observed that these tones appeared to effect components of jet noise emission in such a way that could only be explained by enhanced mixing of the freejet shear layer. A particularly noteworthy experiment was conducted by Lepicovsky et al.,[62] who studied internal nozzle plenum excitation associated with heated high subsonic jets. The results of this study went far to explain the effect of combustor tones on jet noise. Later Tam and Morris[63] provided a theoretical explanation of



a)



b)

**Fig. 24 Mixing enhancement with external excitation: a) sound direction upstream and b) sound direction with flow (from anonymous Russian author, date unknown).**

the effect using instability theory. Goldstein and Leib[64] theoretically examined the effect of external acoustic excitation on free shear layers. They found that mixing could be either enhanced or reduced, depending on the frequency and propagation direction of the sound wave. Figure 24, from the Russian literature (anonymous Russian author, date unknown) illustrates the effect. It can be observed that when sound is propagating against the jet flow mixing is unaltered, but when sound is directed with the flow substantial improvement in jet mixing occurs.

Whether one applies internal or external acoustic excitation to the reacting shear layer for a scramjet, one needs to keep in mind several models proposed by Humphrey and Culick[65] and Sterling and Zukoski,[66] who treated the effect of pressure oscillations on combustion. The research cited earlier was conducted for unexcited shear layers.

### Wavy Wall

The wavy-wall concept, shown in Fig. 25 represents the most efficient way to stimulate growth of large-scale structure and, consequently, mixing efficiency in a supersonic shear layer. In an actual application, the wavy wall would produce weak waves that would interact with the shear layer to excite the most unstable mode. The wavelength structure along the wall would have an increasing wavelength along the wall recognizing that the phase speed of the large-scale structure is dispersive. The amplitude of the wavy-wall structure would be slightly larger than typically expected for material finish in a scramjet engine. At a certain downstream point, however, the wall amplitude would be sufficient to produce a shock wave

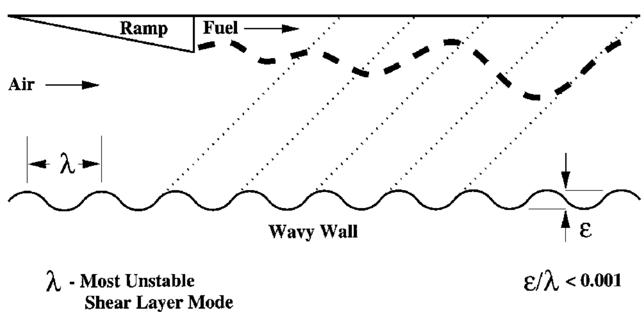



**Fig. 25 Concept for wavy wall excitation of reacting shear layer.**

capable of destroying large-scale structures and promoting small scales needed for high combustion efficiency. Thus, the concept is to excite first for rapid mixing and then generate small scales for combustion after sufficient mixing.

### Flip-Flop Nozzle

This concept is an outgrowth of research on fluidic devices being developed for hybrid computers in the 1960s. Viets[67] applied these concepts to nozzles in an effort to obtain thrust vectoring. Figure 26 shows the ability of this concept to vector a jet. Based on selected geometry, the nozzle flow can be oscillated in any direction at any frequency. The acoustic forcing can be also be used to control the phase of the oscillation. When combined with longitudinal forcing,

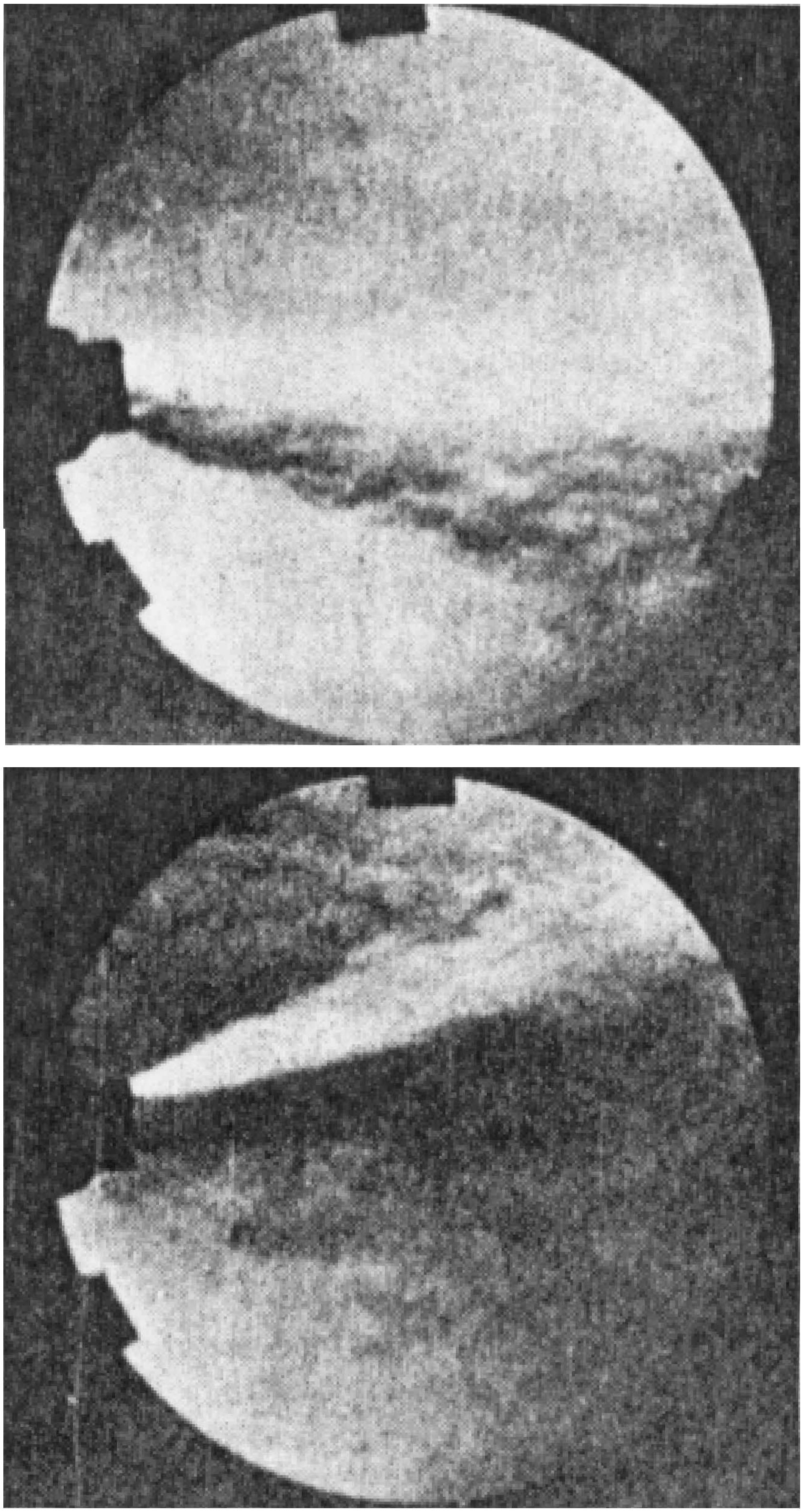


**Fig. 26 Schlieren records of oscillating jet from flip-flop nozzle (from Ref. 67).**

as from the Hartmann–Sprenger tube, the flow can be made to explode in all directions from the nozzle exit.

## Conclusions

Scramjet combustor design requires development of a fuel injection system that is aeroperformance efficient. At the same time, the injector must achieve rapid macroscale mixing of fuel with oxidizer, permit sufficient residence time for fuel heating, destroy large-scale turbulence structure, promote generation of small-scale turbulence for micromixing when sufficient reactants have macromixed, micromix at a rate consistent with chemical reaction times, and control pressure rise due to heat generation.

Whereas combustion takes place on the molecular scale, mixing of fuel and oxidizer takes place at dissipation scales of turbulence. For scramjet applications, methods utilized to increase mixing efficiency of fuel with oxidizer must first be capable of rapidly mixing both supersonic streams on a macroscale and then generating small-scale turbulence to increase diffusion of fuel. The time for rapid mixing should be equal to the time required to heat the fuel to ignition temperature.

The fuel injector concept that appears to be the one of choice at the moment is the ramped fuel injector. This injector does have many of the desirable features discussed. However, current ramp geometry does not have smooth aerodynamic lines, although the design does have flame-holding capability. The ramp design uses two concepts to enhance rapid mixing. These ramps shed streamwise vorticity, which directly energizes transverse scales needed for macroscale mixing. Ramps also induce compression and expansion waves in the flow that lead to the generation of a baroclinic torque on the flow that also energizes macroscale mixing.

If one were to improve the ramp geometry to achieve lower drag, one would diminish the ramp's capability to generate streamwise vorticity. To recover mixing efficiency, however, one could select various concepts discussed concerning the use of port geometry that could also include tabs, chevrons, intermediate origin designs, or counterflow methodology. All of these methods are known to enhance mixing efficiency. Most directly induce streamwise vorticity, but counterflow achieves improved mixing from self-excited resonance and can vector the fuel flow.

It is clear from the previous research conducted concerning the stability of parallel mixing at supersonic speed and the appearance of swept structure that acoustic excitation of the air/fuel shear layer offers great promise. With such techniques, it would be possible to excite the swept structure by an adaptive means to achieve optimized mixing rates. The use of tuned cavities, pulsed fuel jets from the Hartmann–Sprenger tube, piezoelectric actuators, wavy-wall concept, and oscillating fuel jets from the bistable flip-flop nozzle deserve merit for further consideration. Concepts like the wavy wall permit shear layer stimulation over an extended range so that one can excite the dispersive waves in the shear layer.

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